

Lecture 10: The sticky-price monetary model

Open Economy Macroeconomics, Fall 2006

Ida Wolden Bache

October 31, 2006

Macroeconomic models of exchange rate determination

- Macroeconomic models of exchange rate determination
 - Portfolio balance models
 - The monetary approach
 - * Flexible-price model
 - * Sticky-price model
 - New Open Economy Macroeconomics

The flexible price monetary model cont'd.

- Model equations

$$\begin{aligned}p(t) &= s(t) + p^*(t) \\ \dot{s}(t) &= i(t) - i^*(t) \\ m(t) - p(t) &= -\eta i(t) + \kappa y(t),\end{aligned}$$

- The differential equation for the exchange rate

$$\dot{s}(t) = \frac{1}{\eta} s(t) - z(t)$$

where

$$z(t) = \frac{1}{\eta} (m(t) - p^*(t) - \kappa y(t)) + i^*(t)$$

- General solution

$$s(t) = \left[s(t_0) - \int_{t_0}^t z(\tau) \exp\left(-\frac{1}{\eta}(\tau - t_0)\right) d\tau \right] \exp\left(\frac{1}{\eta}(t - t_0)\right)$$

- $s(t_0)$ determined by the requirement that the exchange rate should tend to a strictly positive value when $t \rightarrow \infty$

$$s(t_0) = \int_{t_0}^{\infty} z(\tau) \exp\left(-\frac{1}{\eta}(\tau - t_0)\right) d\tau$$

- Insert into the general solution

$$s(t) = \int_t^{\infty} \left[\frac{1}{\eta} (m(\tau) - p^*(\tau) - \kappa y(\tau)) + i^*(\tau) \right] \exp\left(-\frac{1}{\eta}(\tau - t)\right) d\tau$$

- When the exogenous variables are expected to remain constant the solution simplifies to

$$s = m - p^* - \kappa y + \eta i^*$$

- An announcement of a future increase in the money supply $\Delta m > 0$

$$m(t) = \begin{cases} m_0 & \text{if } t_0 < t < t_1 \\ m_0 + \Delta m & \text{if } t > t_1 \end{cases}$$

– If $t_0 < t < t_1$

$$\begin{aligned} s(t) &= s_0 + \int_{t_1}^{\infty} \frac{1}{\eta} \Delta m \exp\left(-\frac{1}{\eta}(\tau - t)\right) d\tau \\ &= s_0 + \frac{1}{\eta} \Delta m \left(\underbrace{-\eta \exp\left(-\frac{1}{\eta}(\infty - t)\right)}_0 + \eta \exp\left(-\frac{1}{\eta}(t_1 - t)\right) \right) \\ &= s_0 + \Delta m \exp\left(-\frac{1}{\eta}(t_1 - t)\right) \end{aligned}$$

– If $t > t_1$: $s_0 + \Delta m$

- Important lesson: the exchange rate only jumps when new information arrives

- A two-country model:

- Foreign money demand function (assuming $\eta^* = \eta$ and $\kappa^* = \kappa$)

$$m^*(t) - p^*(t) = -\eta i^*(t) + \kappa y^*(t)$$

- Solving for $i^*(t)$

$$i^*(t) = -\frac{1}{\eta} (m^*(t) - p^*(t) - \kappa y^*(t))$$

- Substitute into solution for the exchange rate

$$s(t) = \int_t^\infty \left[\frac{1}{\eta} (m(\tau) - m^*(\tau) - \kappa (y(\tau) - y^*(\tau))) \right] \exp \left(-\frac{1}{\eta} (\tau - t) \right) d\tau$$

- Note: Exchange rate is determinate only in the special case where the domestic and foreign money demand equations have the same elasticities

- General lessons from the flexible-price monetary model
 - the implications of rational expectations for exchange rate dynamics
 - the method of solution
 - the differential effects of temporary and permanent shocks and of anticipated and unanticipated events
 - the importance of relative developments between countries

The sticky-price monetary model (the Dornbusch 'overshooting' model)

- Required readings:
 - Rødseth (2000) chapter 6.7 (+6.1 + appendix A)
 - Obstfeld&Rogoff (1996) chapter 9.1-9.3 (see in particular section on empirical evidence)
- Supplementary readings:
 - Dornbusch, R. (1976): Expectations and exchange rate dynamics. *Journal of Political Economy*, vol. 84, pp. 1161-1176
 - Rogoff, K. (2002): Dornbusch's overshooting model after twenty-five years. *IMF Staff Papers*, vol. 49 (special issue), pp. 1-35

The purpose is to develop a theory that is suggestive of the observed large fluctuations in exchange rates while at the same time establishing that such exchange rate movements are consistent with rational expectations formation.
Dornbusch (1976, p. 1161)

Model equations

- IS-curve

$$Y = C(Y) + X \left(\frac{SP^*}{P}, Y, Y^* \right) \quad (1)$$

where $R = SP^*/P$ is real exchange rate *in terms of producer prices* (= terms of trade) and $0 < C_Y < 1$, $X_R > 0$, $X_Y < 0$, $X_{Y^*} > 0$

- Equilibrium condition in the money market

$$\frac{M}{P} = m(i, Y) \quad (2)$$

where $m_i < 0$, $m_Y > 0$

- Phillips curve

$$\frac{\dot{P}}{P} = \gamma(Y - \bar{Y}) \quad (3)$$

where $\gamma > 0$ and \bar{Y} is the equilibrium (or natural) level of output

- UIP condition with rational expectations (perfect foresight)

$$\frac{\dot{S}}{S} = i - i^* \quad (4)$$

- An aside: The Marshall-Lerner condition

- Definition of net exports (in terms of domestic goods)

$$X(R, Y, Y^*) = Z^*(R, Y^*) - RZ(R, Y)$$

- * $Z_R^* > 0, Z_{Y^*}^* > 0, Z_R < 0, Z_Y > 0$

- * Z is the volume of imports

- * Z^* is the volume of exports.

- Effect of real depreciation ($R \uparrow$)

$$X_R = \frac{dX}{dR} = \underbrace{Z_R^* - RZ_R}_{\text{quantity effect}} \quad \underbrace{-Z}_{\text{direct price effect}} \quad \geq 0$$

- Assuming that trade is balanced initially ($Z^* = RZ$) then

$$\begin{aligned}\frac{dX}{dR} &= Z \left(\frac{Z_R^*}{Z} - \frac{RZ_R}{Z} - 1 \right) \\ &= Z \left(\frac{RZ_R^*}{Z^*} - \frac{RZ_R}{Z} - 1 \right) \\ &= Z (El_R Z^* + El_R Z - 1)\end{aligned}$$

where $El_R Z^*$ and $El_R Z$ are the (absolute values) of the price elasticities of export and import demand, respectively.

- Necessary and sufficient condition for a real depreciation to have a positive effect on the trade balance:

$$El_R Z^* + El_R Z > 1$$

which is the *Marshall-Lerner condition*

- In the following: assume $X_R > 0$

- Back to the model...
- Classification of variables:
 - Endogenous variables Y, i, P and S .
 - Exogenous variables i^*, Y^*, M, P^* and \bar{Y}
- Initial value of P (P_0) is given. P is a “crawler” variable and change only gradually over time
- S is a “jump” variable: the initial value of S has to be determined endogenously

Solving the model

- First step: Derive a system of first-order non-linear differential equations in S and P
- For given values of P and S , equations (1) and (2) define a temporary equilibrium for Y and i :

$$Y = Y\left(\frac{SP^*}{P}, Y^*\right) \quad (5)$$

$$i = i\left(\frac{M}{P}, \frac{SP^*}{P}, Y^*\right) \quad (6)$$

- Total differentiation of

$$Y = C(Y) + X \left(\frac{SP^*}{P}, Y, Y^* \right)$$

yields

$$dY = C_Y dY + X_R \frac{P^*}{P} dS - X_R \frac{SP^*}{P^2} dP + X_R \frac{S}{P} dP^* + X_Y dY + X_{Y^*} dY^*$$

- Implicit derivatives

$$\frac{dY}{dP} = - \frac{SP_*}{P^2} \frac{X_R}{1 - C_Y - X_Y} < 0$$

$$\frac{dY}{dS} = \frac{P^*}{P} \frac{X_R}{1 - C_Y - X_Y} > 0$$

$$\frac{dY}{dP^*} = \frac{S}{P} \frac{X_R}{1 - C_Y - X_Y} > 0$$

$$\frac{dY}{dY^*} = \frac{X_{Y^*}}{1 - C_Y - X_Y} > 0$$

- Total differentiation of

$$\frac{M}{P} = m(i, Y)$$

yields

$$\frac{1}{P}dM - \frac{M}{P^2}dP = m_i di + m_Y dY$$

- Implicit derivatives

$$\begin{aligned} \frac{di}{dP} &= -\frac{m_Y dY}{m_i dP} - \frac{M}{P^2} \frac{1}{m_i} \\ &= \frac{m_Y SP^*}{m_i P^2} \frac{X_R}{1 - C_Y - X_Y} - \frac{M}{P^2} \frac{1}{m_i} \\ &= \frac{M}{P^2} \frac{1}{m_i} \left(\frac{m_Y}{M} SP^* \frac{X_R}{1 - C_Y - X_Y} - 1 \right) \\ &= \frac{M}{P^2} \frac{1}{m_i} \left(\frac{PY}{M} m_Y \frac{SP^*}{P} \frac{1}{Y} \frac{X_R}{1 - C_Y - X_Y} - 1 \right) \\ &= \frac{M}{P^2} \frac{1}{m_i} \left(El_Y \left(\frac{M}{P} \right) \times El_{RY} - 1 \right) \stackrel{\leq}{\geq} 0 \end{aligned}$$

- The sign of di/dP is ambiguous!
- Overshooting requires that $El_Y \left(\frac{M}{P} \right) \times El_R Y < 1$ (i.e., $di/dP > 0$)

- More derivatives

$$\begin{aligned} \frac{di}{dS} &= -\frac{m_Y dY}{m_i dS} \\ &= -\frac{m_Y P^*}{m_i P} \frac{X_R}{1 - C_Y - X_Y} > 0 \\ \frac{di}{dM} &= \frac{1}{m_i P} < 0 \\ \frac{di}{dY_*} &= -\frac{m_Y dY}{m_i dY^*} \\ &= -\frac{m_Y}{m_i} \frac{X_{Y_*}}{1 - C_Y - X_Y} > 0 \\ \frac{di}{dP^*} &= -\frac{m_Y dY}{m_i dP^*} \\ &= -\frac{m_Y S}{m_i P} \frac{X_R}{1 - C_Y - X_Y} > 0 \end{aligned}$$

- Inserting (5) and (6) into (3) and (4) we get the two differential equations

$$\dot{P} = \phi_1(P, S; P^*, Y^*) \quad (7)$$

$$\dot{S} = \phi_2(P, S; P^*, Y^*, M, i^*) \quad (8)$$

- The stationary equilibrium is defined by

$$\dot{P} = 0 \Leftrightarrow Y\left(\frac{SP^*}{P}, Y^*\right) = \bar{Y} \text{ (determines } R) \quad (9)$$

$$\dot{S} = 0 \Leftrightarrow i\left(\frac{M}{P}, \frac{SP^*}{P}, Y^*\right) = i^* \text{ (determines } P, \text{ and hence } S) \quad (10)$$

Solving systems of two differential equations: general case

- Simple homogenous linear system

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2$$

- Stationary equilibrium: $(x_1 = 0, x_2 = 0) \rightarrow \dot{x}_1 = \dot{x}_2 = 0$

- Define

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

- Recall that

$$\text{tr}(A) = a_{11} + a_{22}$$

$$|A| = a_{11}a_{22} - a_{12}a_{21}$$

- Definition of stability:
 - The system is stable if for arbitrary initial values x_1 and x_2 tend to the stationary equilibrium as $t \rightarrow \infty$
 - The system exhibits *saddle path stability* if there is a unique convergent path to the steady state. That is; if the initial value of one of the variables is given, there is a unique initial value of the other variable consistent with the system converging to the stationary equilibrium as $t \rightarrow \infty$
- Mathematical conditions for stability
 - Iff $tr(A) < 0$ and $|A| > 0$ the system is stable
 - If $|A| < 0$ the system exhibits *saddle path stability*